Effect of a Rotor on the Attitude Stability of a Satellite in a Circular Orbit

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In the gravitational field of a fixed particle, the mass center of an unsymmetrical rigid body can move on a circular orbit centered at the particle while one of the centroidal principal axes of inertia of the body remains normal to the plane of this orbit. At the same time, a second centroidal principal axis can either rotate relative to the line joining the particle to the mass center of the body, or it can oscillate about this line as a mean position. The stability of such motions has been discussed previously. The present paper deals with the following question: How does the inclusion of a symmetrical rotor affect the stability of the orbiting body? To answer this question, a general procedure for testing the stability of the motions under consideration is developed, and this procedure is used to examine a number of specific cases in detail. The results show that both beneficial and harmful effects can be produced rather easily; that is, that light, low-speed rotors can act either as stabilizers or as destabilizers.

Introduction

A RECENT paper[1] in this Journal dealt with the stability of motion of a rigid body carrying a disk that can be made to rotate about an axis fixed in the body. As anticipated by Roberson[2] as long ago as 1957, the results indicated that torques brought into play by the disk have a substantial effect on the attitude motions of the main body, and the problem is thus of immediate interest in connection with satellites. However, as gravitational effects were not taken into account in this work, and as a number of recent papers have shown that attitude motions of both a single rigid body[3–6] and of various gyroscopic devices[7] are profoundly affected by gravitational forces, the need for further investigation is clearly indicated. Such an investigation is the subject of the present paper.

Specifically, a satellite consisting of two parts, one a rigid body R having no special symmetries, the other a rigid body R' possessing a centroidal axis of inertial symmetry, is considered. R' is constrained to rotate about its axis of symmetry, which coincides with one of the centroidal principal axes of inertia of R, and the mass center of the satellite is presumed to move on a circular orbit whose center coincides with an attracting particle representing the earth for the purpose of evaluating gravitational forces. Attention is focused on motions during which the symmetry axis of R' remains in the neighborhood of the normal to the plane of this orbit.

The paper is divided into three parts, "Dynamics," "Stability," and "Results." The first part contains a brief derivation of the differential equations governing the attitude behavior of the satellite. In the second part, Floquet theory is used to develop a procedure for testing the stability of the motions under consideration, and this procedure is used in the third part to examine in detail particular cases that are of interest either because they serve as a check on the procedure or because they lead to conclusions having practical implications. For example, the feasibility of stabilization by means of a very light, low-speed rotor is demonstrated, as is the dangerous concomitant possibility of destabilization. It thus becomes evident that, in the design of certain satellite systems, the relationship between attitude stability and system parameters must be studied with considerable care. The procedure developed in this paper should facilitate such studies, even when they are based on more elaborate mathematical models, such as those accounting for the earth's oblateness, atmospheric resistance, internal energy dissipation, etc. (With regard to energy dissipation it is important to keep in mind that this can affect stability significantly, as pointed out most recently by Zajac.[8])

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Dynamics

In Fig. 1, R and R' designate the two rigid bodies comprising the satellite under consideration. P^* is the mass center of each of the bodies, and thus of the satellite, and X_1, X_2, and X_3 are principal axes of inertia of R, the corresponding moments of inertia being I_1, I_2, and I_3. X_1', X_2', X_3' are mutually perpendicular lines fixed in body R', and X_3' coincides with X_3, so that the only possible motions of R' relative to R are rotations about this common axis. Furthermore, X_1', X_2', and X_3' are principal axes of R', and two of the associated moments of inertia, I_1' and I_2' are taken to be equal to each other. Finally, ψ measures the angle between X_1 and X_1', and it is assumed that R' is driven relative to R (by a motor not shown) in such a way that $d\psi/dt = s$, a constant, called the "spin."

Orbital reference axes A_1, A_2, and A_3 are shown in Fig. 2. The line passing through the center of the earth and the mass center P^* of the satellite is A_1; A_2 is tangent to the assumed

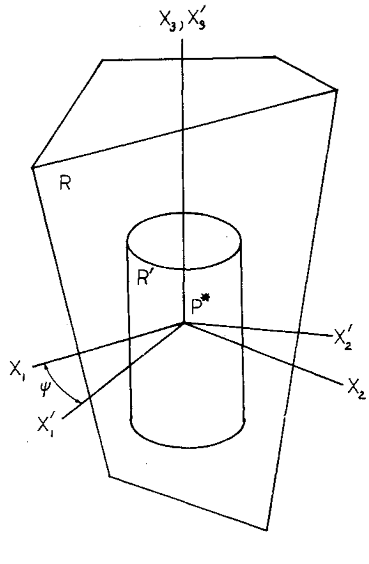

Fig. 1 Schematic representation of the satellite.

circular orbit; and A_3 is perpendicular to the orbital plane. Any desired orientation of the satellite relative to these axes can be produced by first aligning X_i with A_i, $i = 1, 2, 3$; next, performing a right-handed rotation of R of amount θ_1 about A_1, bringing X_i into coincidence with B_i, $i = 1, 2, 3$; and following this with rotations of amount θ_2 about B_2, leading to C_1, C_2, C_3, and θ_3 about C_3, bringing R into its final position.

The angle ϕ between A_3, the normal to the orbital plane, and the body-fixed axis X_3 depends on θ_1 and θ_2, but not on θ_3. Consequently, expressions used to study motions during which X_3 remains nearly aligned with A_3 shall be linearized in θ_1 and θ_2. For example, the angular velocity of R in inertial space, referred to the axes X_1, X_2, and X_3, is then given by

$$\left.\begin{aligned} \omega_1 &= (\dot\theta_2 + \Omega\theta_1)\sin\theta_3 + (\dot\theta_1 - \Omega\theta_2)\cos\theta_3 \\ \omega_2 &= (\dot\theta_2 + \Omega\theta_1)\cos\theta_3 - (\dot\theta_1 - \Omega\theta_2)\sin\theta_3 \\ \omega_3 &= \dot\theta_3 + \Omega \end{aligned}\right\} \quad (1)$$

where Ω is the (constant) "orbital angular speed"; similarly, the sum of the moments about P^* of the gravitational forces exerted on the satellite by the earth, the so-called "gravity torque," also referred to the axes X_1, X_2, and X_3, is[9]

$$\left.\begin{aligned} M_1 &= 3\Omega^2(\bar I_2 - \bar I_3)\theta_2\sin\theta_3 \\ M_2 &= 3\Omega^2(\bar I_1 - \bar I_3)\theta_2\cos\theta_3 \\ M_3 &= 3\Omega^2(\bar I_1 - \bar I_2)\sin\theta_3\cos\theta_3 \end{aligned}\right\} \quad (2)$$

where $\bar I_1$, $\bar I_2$, and $\bar I_3$ are the moments of inertia of the satellite about the axes X_1, X_2, and X_3, respectively, that is,

$$\bar I_i = I_i + I_i' \qquad i = 1, 2, 3 \quad (3)$$

The angular-momentum principle, applied to the entire (nonrigid) satellite, requires that

$$\bar I_1\dot\omega_1 - \omega_2\omega_3(\bar I_2 - \bar I_3) + \omega_2 s I_3' = M_1 \quad (4)$$

$$\bar I_2\dot\omega_2 - \omega_3\omega_1(\bar I_3 - \bar I_1) - \omega_1 s I_3' = M_2 \quad (5)$$

$$\bar I_3\dot\omega_3 - \omega_1\omega_2(\bar I_1 - \bar I_2) = M_3 \quad (6)$$

The analysis that follows is facilitated by the introduction of the dimensionless parameters

$$\left.\begin{aligned} \bar K_1 &= (\bar I_2 - \bar I_3)/\bar I_1 \\ \bar K_2 &= (\bar I_3 - \bar I_1)/\bar I_2 \end{aligned}\right. \quad (7)$$

$$\left.\begin{aligned} J_1 &= (I_3'/\bar I_1)(s/\Omega) \\ J_2 &= (I_3'/\bar I_2)(s/\Omega) \end{aligned}\right. \quad (8)$$

and the frequency-like quantity $\bar p$ defined by

$$\bar p^2 = 3\Omega^2(\bar K_1 + \bar K_2)/(1 + \bar K_1\bar K_2) \quad (9)$$

Furthermore, it is convenient to eliminate the independent variable t by means of the substitution

$$t = \tau/\Omega \quad (10)$$

and to use four variables x_1, x_2, x_3, and x_4 in place of θ_1, θ_2, and the derivatives of θ_1 and θ_2 with respect to τ, that is, to let

$$x_1 = \theta_1 \qquad x_2 = \theta_2 \quad (11)$$

$$x_3 = x_1' \qquad x_4 = x_2' \quad (12)$$

where primes denote differentiation with respect to τ. Substitution from (1) and (2) into (6) then gives

$$\theta_3'' + (\bar p/\Omega)^2 s_3 c_3 = 0 \quad (13)$$

where s_3 and c_3 are abbreviations for $\sin\theta_3$ and $\cos\theta_3$, respectively; and (4, 5, and 1) lead to the system of four first-order differential equations

$$x_i' = \sum_{j=1}^{4} w_{ij}x_j \qquad i = 1, 2, 3, 4 \quad (14)$$

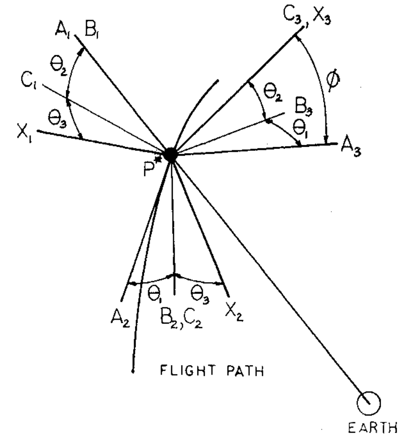

Fig. 2 Reference axes and attitude angles.

where

$$\left.\begin{aligned}
&w_{11} = 0 \qquad w_{12} = 0 \qquad w_{13} = 1 \qquad w_{14} = 0 \\
&w_{21} = 0 \qquad w_{22} = 0 \qquad w_{23} = 0 \qquad w_{24} = 1 \\
&w_{31} = [\bar K_1 - J_1 - (1 - \bar K_1)\theta_3']c_3^2 - [\bar K_2 + J_2 + (1 + \bar K_2)\theta_3']s_3^2 \\
&w_{32} = [4(\bar K_1 + \bar K_2) - J_1 + J_2 + (\bar K_1 + \bar K_2)\theta_3']s_3c_3 \\
&w_{33} = -[\bar K_1 + \bar K_2 - J_1 + J_2 + (\bar K_1 + \bar K_2)\theta_3']s_3c_3 \\
&w_{34} = [1 + \bar K_1 - J_1 - (1 - \bar K_1)\theta_3']c_3^2 + [1 - \bar K_2 - J_2 - (1 + \bar K_2)\theta_3']s_3^2 \\
&w_{41} = -w_{33} \\
&w_{42} = [4\bar K_1 - J_1 - (1 - \bar K_1)\theta_3']s_3^2 - [4\bar K_2 + J_2 + (1 + \bar K_2)\theta_3']c_3^2 \\
&w_{43} = -[1 + \bar K_1 - J_1 - (1 - \bar K_1)\theta_3']s_3^2 - [1 - \bar K_2 - J_2 - (1 + \bar K_2)\theta_3']c_3^2 \\
&w_{44} = w_{41}
\end{aligned}\right\} \quad (15)$$

Before proceeding to the discussion of stability, it is worth mentioning that the system of Eqs. (14) possesses an integral similar to that described by Zajac.[11] Whereas this integral cannot be used directly to obtain stability conditions in the present problem, it can prove helpful in other connections, e.g., for a full integration of the equations of motion.

Stability

When θ_1 and θ_2 vanish identically, it follows from (11) that x_1, x_2, x_3, x_4 also vanish for all values of τ and that Eqs. (14) are thus satisfied regardless of the behavior of θ_3. It is the stability of motions such as these that is to be examined [stability here meaning that, by taking θ_1, θ_2, $\dot\theta_1$, and $\dot\theta_2$ sufficiently small (but not equal to zero) for $t = t_0$, one can keep θ_1 and θ_2 arbitrarily small for $t \geq t_0$.] Now, as (13) can be recognized to be the differential equation of motion of a pendulum, it is evident that θ_3 can be either an oscillatory or monotone function of τ, according as $|\theta_3'|$ is larger or smaller than $\bar p/\Omega$ when θ_3 is equal to zero. In the first case, hereafter referred to as "oscillatory," θ_3 is a periodic function of τ, the period τ_0 being given by

$$\tau_0 = 4(\Omega/\bar p)K(k_0) \quad (16)$$

where K is the complete elliptic integral of the first kind, and the modulus k_0 of K can be expressed both in terms of the amplitude θ_3^* of the oscillations,

$$k_0 = \sin\theta_3^* \quad (17)$$

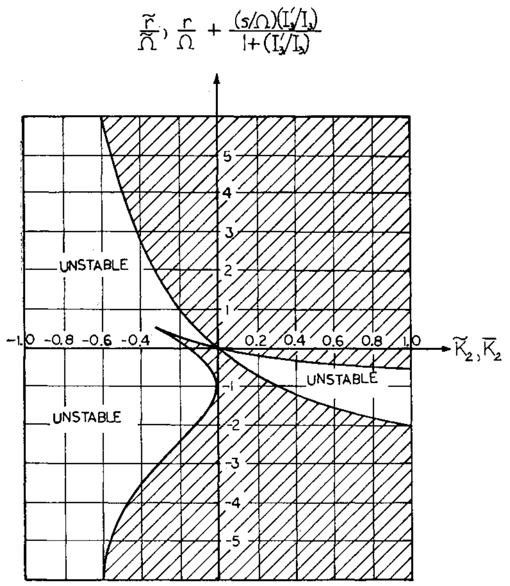

Fig. 3 Instability chart of a symmetric satellite.

or as a function of the value r of $\dot{\theta}_3$ when θ_3 is equal to zero,

$$k_0 = (r/\bar{p})\ \mathrm{sgn}(r) \tag{18}$$

In the second, or "rotational" case, θ_3 is not a periodic function, but there exists a quasi-period τ_r defined as the change in τ corresponding to a change of 2π rad in θ_3:

$$\tau_r \equiv 4(\Omega/\bar{p})k_rK(k_r) \tag{19}$$

where

$$k_r = (\bar{p}/r)\ \mathrm{sgn}(r) \tag{20}$$

An analog to (17) can be obtained in this case by introducing a quantity α defined as

$$\alpha \equiv (2\pi/\tau_r)\ \mathrm{sgn}(r) \tag{21}$$

to provide a measure of the average rotational speed of the body R relative to the orbiting reference axes A_1, A_2, and A_3. Elimination of τ_r and $\Omega/\bar{p}$ from (19) by use of (9) and (21) then gives

$$k_rK(k_r) = \frac{\pi}{2\alpha}\left(3\,\frac{\bar{K}_1 + \bar{K}_2}{1 + \bar{K}_1\bar{K}_2}\right)^{1/2} \mathrm{sgn}(r) \tag{22}$$

and it may be noted for future reference that

$$\mathrm{sgn}(r) = \mathrm{sgn}(\alpha) \tag{23}$$

In the sequel it becomes important that Eqs. (14) are differential equations with periodic coefficients in both the oscillatory and rotational cases. This may be seen as follows.

Equation (13) possesses the first integral

$$(\theta_3')^2 + (\bar{p}/\Omega)^2 \sin^2\theta_3 = \mathrm{const}$$

Consequently, $\theta_3'(\tau)$ is a periodic function of τ with the same period as $\sin[\theta_3(\tau)]$. Now, for the oscillatory case, τ_0 was defined in such a way that

$$\theta_3(\tau + \tau_0) = \theta_3(\tau)$$

so that

$$\sin[\theta_3(\tau + \tau_0)] = \sin[\theta_3(\tau)]$$

whereas, in ther otating case, as a consequence of the definition of τ_r,

$$\theta_3(\tau + \tau_r) = \theta_3(\tau) + 2\pi$$

$$\sin[\theta_3(\tau + \tau_r)] = \sin[\theta_3(\tau) + 2\pi] = \sin[\theta_3(\tau)]$$

$\theta_3'(\tau)$ is thus a periodic function of τ in both cases, and, in view of (15), this means that Eqs. (14) are linear differential equations with periodic coefficients.

In accordance with Floquet theory,[10] the boundedness of the solutions of these equations depends on the value at τ_0, or τ_r, of the 4×4 matrix $H(\tau)$ defined by the matrix differential equation

$$H'(\tau) = w(\tau)H(\tau) \tag{24}$$

and the initial conditions

$$H(0) = I$$

where $w(\tau)$ is the 4×4 matrix with w_{ij} [see (15)] as the element in the ith row and the jth column, and I is the 4×4 unit matrix. Specifically, all solutions of (14) are bounded as $\tau \to \infty$ if and only if the modulus of each of the four characteristic values of $H(\tau_0)$ [or $H(\tau_r)$] is less than or equal to unity, and if, for any characteristic value λ_k such that $|\lambda| = 1$, the multiplicity of λ_k is equal to the nullity of the matrix $H(\tau_0) - \lambda_kI$ [or $H(\tau_r) - \lambda_kI$]. Given the inertia properties of R and R', the ratio s/Ω of the spin to the orbital angular speed, and either θ_3* or α, one thus proceeds as follows:

1) Use (3, 7, and 8) to evaluate $\bar{K}_1$, $\bar{K}_2$, J_1, and J_2.
2) Find k_0 from (17), or k_r from (22) and (23).
3) Determine r by means of (18) and (9), or (20, 9, and 23).
4) Evaluate τ_0 by using (16) and (9), or τ_r from (21) and (23).
5) Perform simultaneously a numerical (digital computer) integration of (13) and the 16 equations (24), using for initial values $\theta_3(0) = 0$, $\theta_3'(0) = r/\Omega$ [see (18) and (20) for r], and $H(0) = I$, and terminating the integration at $\tau = \tau_0$, or $\tau = \tau_r$.
6) Find the roots λ_1, λ_2, λ_3, and λ_4 of the characteristic equation

$$\det[H(\tau_0) - \lambda I] = 0$$

or

$$\det[H(\tau_r) - \lambda I] = 0$$

and determine the modulus of each distinct root. If any of these exceeds unity, or if it is equal to unity and the multiplicity of the corresponding root λ_k is not equal to the nullity of the matrix $H(\tau_0) - \lambda_kI$, or $H(\tau_r) - \lambda_kI$, then the motion $\theta_1 = \theta_2 = 0$ is unstable. When these requirements for instability are not fulfilled, stability is indicated, but not assured, because Eqs. (14) were obtained by linearization.

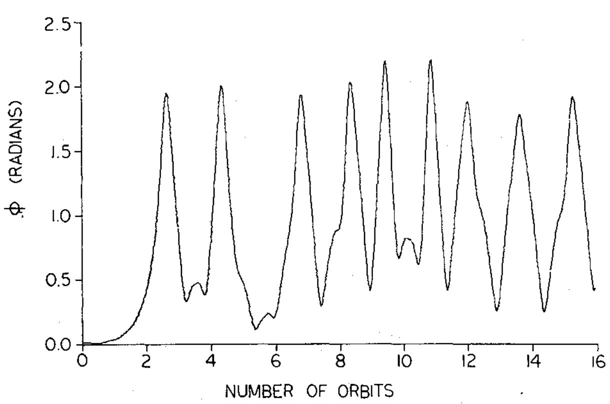

Fig. 4 Behavior of R alone; $\theta_1(0) = \theta_2(0) = 0.01$ rad.

Results

The moments of inertia I_1' and I_2' of the body R' were taken equal to each other at the outset. If I_1 and I_2 are also set equal to each other, there result situations that are not only of interest in their own right, but can also serve as a check on the procedure described in the preceding section, for the differential equations of motion then become equations with constant coefficients, which makes it possible to discuss stability without reference to this procedure. Better than that, one can make direct use of the results obtained by Thomson.[5, 6] This may be seen as follows. When $I_1 = I_2$ and $I_1' = I_2'$, it follows from (3) that $\bar{I}_1 = \bar{I}_2$, and hence from (7) that

$$\bar{K}_1 = -\bar{K}_2 \tag{25}$$

and from (9) that $\bar{p} = 0$. In view of (13), this means that θ_3' remains constant (and therefore equal to its initial value) throughout the motion. Consequently,

$$\theta_3' = r/\Omega \tag{26}$$

where, as before, r is the value of $\dot{\theta}_3$ when θ_3 is equal to zero.

Substitution from (25) and (26) into (15) leaves w_{1i} and w_{2i}, $i = 1, 2, 3, 4$, unaltered and leads to the following (constant) expressions for w_{3i} and w_{4i}, $i = 1, 2, 3, 4$:

$$\left.\begin{aligned} w_{31} &= -[J_1 + \bar{K}_2 + (1 + \bar{K}_2)(r/\Omega)] \\ w_{34} &= 1 + w_{31} \\ w_{42} &= -[J_1 + 4\bar{K}_2 + (1 + \bar{K}_2)(r/\Omega)] \\ w_{43} &= -w_{34} \\ W_{32} &= w_{33} = w_{41} = w_{44} = 0 \end{aligned}\right\} \tag{27}$$

Suppose now that the body R' were absent. Then the present problem would reduce to that treated by Thomson.[5] Let $\tilde{R}$ designate R under these circumstances, and let $\tilde{r}$, $\tilde{\Omega}$, etc. be the values of r, Ω, etc. required for the description of the motion of $\tilde{R}$. Then relations corresponding to (27) can be obtained by simply deleting J_1 [see (8)] and by replacing r, Ω, and $\bar{K}_2$ [see (7)] with $\tilde{r}$, $\tilde{\Omega}$, and $\tilde{K}_2$, respectively. In particular,

$$\tilde{w}_{31} = -[\tilde{K}_2 + (1 + \tilde{K}_2)(\tilde{r}/\tilde{\Omega})]$$

$$\tilde{w}_{42} = -[4\tilde{K}_2 + (1 + \tilde{K}_2)(\tilde{r}/\tilde{\Omega})]$$

To devise a body $\tilde{R}$ and a motion of this body with the same stability characteristics as the system under consideration, it is, thus, merely necessary to choose $\tilde{r}$, $\tilde{\Omega}$, and $\tilde{K}_2$ such that

$$J_1 + \bar{K}_2 + (1 + \bar{K}_2)(r/\Omega) = \tilde{K}_2 + (1 + \tilde{K}_2)(\tilde{r}/\tilde{\Omega})$$

and

$$J_1 + 4\bar{K}_2 + (1 + \bar{K}_2)(r/\Omega) = 4\tilde{K}_2 + (1 + \tilde{K}_2)(\tilde{r}/\tilde{\Omega})$$

which is readily accomplished by taking

$$\tilde{K}_2 = \bar{K}_2 \tag{28}$$

and

$$\tilde{r}/\tilde{\Omega} = r/\Omega + J_1/(1 + \bar{K}_2)$$

or, in view of (3, 7, and 8), and keeping in mind that $I_1 = I_2$,

$$\frac{\tilde{r}}{\tilde{\Omega}} = \frac{r}{\Omega} + \frac{s}{\Omega}\,\frac{I_3'/I_3}{1 + (I_3'/I_3)} \tag{29}$$

The stability chart for a single rigid body[6] thus becomes applicable to the present problem as soon as the axes have been renamed in accordance with (28) and (29), as shown in Fig. 3.

As one of the axes in Fig. 3 is labeled $\tilde{K}_2$, it is worth noting that this quantity can be expressed in terms of three inertia ratios, each of which has a clear-cut physical significance. These are I_3/I_1, I_3'/I_1', and I_3'/I_3, the first two of which characterize the bodies R and R', respectively, whereas the third relates the two bodies to each other. Substitution from (3) into the second of Eqs. (7) gives

$$\bar{K}_2 = \frac{1 + (I_3'/I_3) - [1/(I_3/I_1)] - [(I_3'/I_3)/(I_3'/I_1')]}{[1/(I_3/I_1)] + [(I_3'/I_3)/(I_3'/I_1')]} \tag{30}$$

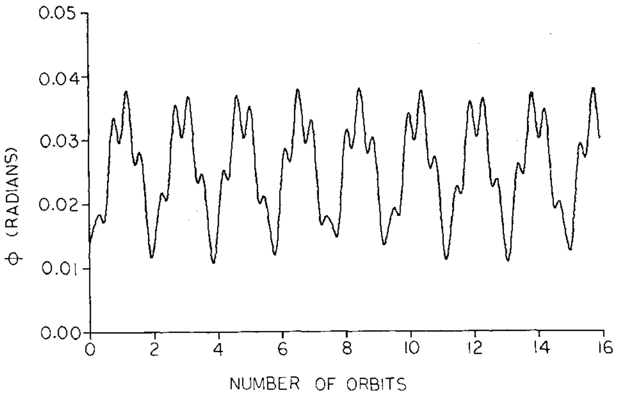

Fig. 5 Behavior of R with R'; $\theta_1(0) = \theta_2(0) = 0.01$ rad.

To illustrate the use of these results, we let R be a solid right-circular cylinder of equal radius and height, so that

$$I_3/I_1 = 1.5 \tag{31}$$

and take for R' a thin circular disk, whence

$$I_3'/I_1' = 2.0 \tag{32}$$

Furthermore, we set

$$I_3'/I_3 = 0.01 \tag{33}$$

thus making R the "main" body, and now seek to determine values of the spin s such that R' stabilizes R in an attitude that remains fixed in inertial space, i.e., an attitude for which $r/\Omega = -1$. (That a stabilizer is required may be seen from the fact that, in the absence of R', and with $I_1 = I_2$,

$$\bar{K}_2 = \frac{I_3 - I_1}{I_2} = \frac{I_3}{I_1} - 1 = 0.5$$

and the point $\tilde{K}_2 = 0.5$, $\tilde{r}/\tilde{\Omega} = -1$ lies in a zone labeled unstable in Fig. 3.)

The computational procedure described in the preceding section, applied for values of s/Ω lying between -100 and $+100$, predicts instability for

$$-34 < s/\Omega < 67 \tag{34}$$

With $r/\Omega = -1$ and $I_3'/I_3 = 0.01$, this means that

$$-1.337 < \frac{r}{\Omega} + \frac{s}{\Omega}\,\frac{I_3'/I_3}{1 + (I_3'/I_3)} < -0.337 \tag{35}$$

Now, from (30) together with (31–33),

$$\bar{K}_2 = 0.504 \tag{36}$$

Consequently, (35) should describe that portion of the line

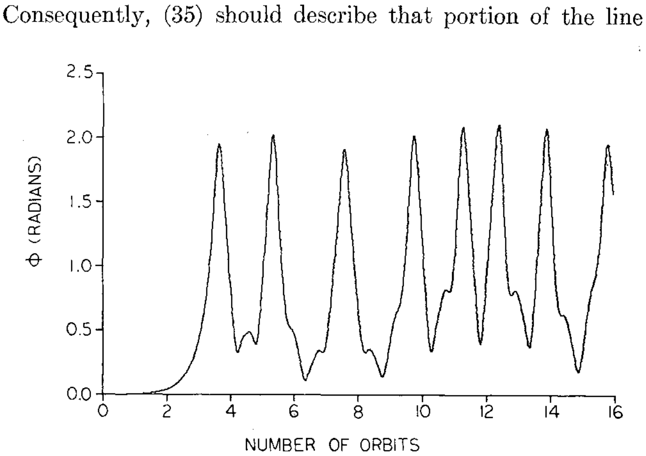

Fig. 6 Behavior of R alone; $\theta_1(0) = \theta_2(0) = 0.001$ rad.

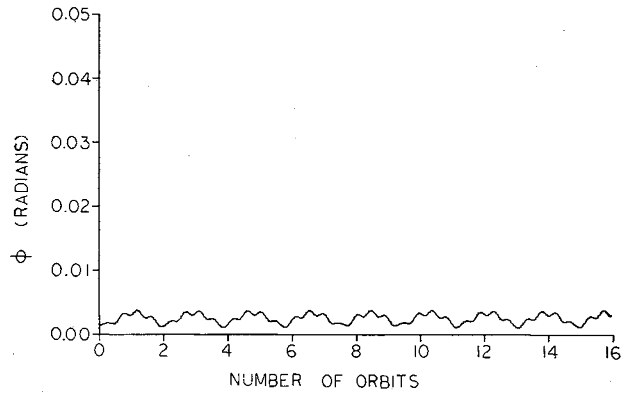

Fig. 7 Behavior of R with R'; $\theta_1(0) = \theta_2(0) = 0.001$ rad.

$\bar{K}_2 = 0.504$ in Fig. 3 which lies in a zone designated as unstable, and this is seen to be the case.

From (34) it appears that R' might serve as a stabilizer for R either if $s/\Omega < -34$ or if $s/\Omega > 67$. Some of the negative values of s/Ω satisfying this requirement lead to smaller absolute values of s than do any of the positive values. In practical applications, this is of interest because it has a direct bearing on questions of energy consumption. Hence, in choosing a specific value to demonstrate the stabilizing effect of R' on R, we take $s/\Omega = -50$ and integrate the full (nonlinear) set of equations of motion, using the following initial values to simulate a disturbance of the motion whose stability is under consideration:

$$\theta_1(0) = \theta_2(0) = 0.01 \qquad \dot{\theta}_1(0) = \dot{\theta}_2(0) = 0$$

$$\theta_3(0) = 0 \qquad \dot{\theta}_3(0) = -\Omega$$

In Figs. 4–7, the results of such integrations are shown in the form of plots of the angle ϕ between the body-fixed axis X_3 and the normal to the orbit plane A_3. Figure 4 describes the behavior of R in the absence of R', whereas Fig. 5 deals with both bodies. In the first case, θ is seen to grow to about 100 times its initial value in less than three orbits (clearly an indication of instability), whereas, in the second case, the maximum value of ϕ is always less than three times as large as the initial value. Figures 6 and 7 demonstrate the effect of a change in initial conditions, and thus highlight the difference between stable and unstable performance. Figure 6 shows that, when the initial values of θ_1 and θ_2 are reduced by a factor of ten, the maximum value of ϕ, although attained more slowly than before, is numerically unaltered for R in the absence of R' (compare with Fig. 4). By way of contrast, the maxima in Fig. 7 (R and R' together) are far smaller than those in Fig. 5. It is noteworthy that this marked difference in behavior is attributable to a relatively "light" ($I_3'/I_3 = 0.01$) and "low-speed" rotor (50 rev of R' relative to R per orbit).

Turning now to motions involving unsymmetrical bodies R, that is, bodies for which $I_1 \neq I_2$, and considering oscillatory cases first, the effect of R' on R can be discussed best in the light of results obtained previously.[3] For example, it has been determined that, in the absence of R', oscillatory motion

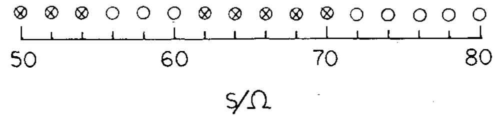

Fig. 8 Effect of rotor spin on stability (X-unstable).

of R with $\theta_2^* = 5°$ is unstable if

$$\frac{I_2 - I_3}{I_1} = -0.50 \qquad \frac{I_3 - I_1}{I_2} = 0.90$$

We now find that this motion can be stabilized by adding R' in the form of a thin, circular disk for which I_3'/I_3 and s/Ω have the modest values 0.01 and 2, respectively. However, the fact that stabilization can be accomplished so easily should not be regarded as an unmixed blessing, for it suggests that destabilization may be produced with equal ease, and this turns out to be the case. For example, if the value of $(I_3 - I_1)/I_2$ is changed from 0.90 to 0.95, and $I_3'/I_3 = 0.01$ while $s/\Omega = 0.5$, the motion under consideration is stable for R alone, but unstable for R and R' together.

Finally, a somewhat different sort of sensitivity of the system may be illustrated in connection with rotational cases. For instance, in the absence of R', the values $(I_2 - I_3)/I_1 = -0.40$, $(I_3 - I_1)/I_2 = 0.75$, and $\alpha = -1.0$ lead to instability.[4] If R' is now once again taken to be a thin, circular disk such that $I_3'/I_3 = 0.01$, and values lying between 50 and 80 are assigned to s/Ω, one obtains Fig. 8, where each circle represents an application of the procedure described in the previous section and crosses indicate instability. It now appears, perhaps surprisingly, that R' may be expected to act as a stabilizer for $56 \leq s/\Omega \leq 60$, but will fail to do so for $62 \leq s/\Omega \leq 70$. In other words, an increase in s/Ω may be deleterious.

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